AP Calculus (AB) Summer Assignment

Please complete each problem **in order** on a separate sheet(s) of paper. This assignment is due about one week into next school year, **not** the first day of school. At that time, you will be asked to submit all of your work and copy several of your answers (**only** the answers) onto an Answer Sheet.

This assignment is tough – there's no other way to say it. You are going to be asked to review all of the algebra and geometry skills that you've ever been taught – and that's a lot! Please don't get too discouraged if you have some trouble; it is to be expected. Also, expect to work for a minimum of 6 hours on this assignment over the summer – **plan accordingly!** You are welcome to use any printed resource at your disposal (old notes, our textbook, Internet sites, etc.) and everyone will probably need to look up something (distance formula, quadratic formula, etc.) – please do so! This assignment, however, is designed to assess if **you** are ready, not if your friends are ready for Calculus, so please understand that it is hoped that you will "do the right thing" and work mostly by yourself. Your Calculus teacher next year is another possible source of help, after you've already tried a problem, but couldn't figure it out.

Good luck! You can do it!!

1. Simplify: (a) $\dfrac{x^3-9x}{x^2-7x+12}$ (b) $\dfrac{x^2-2x-8}{x^3+x^2-2x}$ (c) $\dfrac{\frac{1}{x}-\frac{1}{5}}{\frac{1}{x^2}-\frac{1}{25}}$ (d) $\dfrac{9-x^{-2}}{3+x^{-1}}$

2. Rationalize each denominator: (a) $\dfrac{4}{\sqrt{5}}$ (b) $\dfrac{4}{1-\sqrt{5}}$ (c) $\dfrac{2}{\sqrt{3}+\sqrt{2}}$

3. Write each of the following expressions in the form ca^pb^q where c, p, and q are numbers:

   (a) $\dfrac{2a^2\ ^3}{b}$ (b) $\sqrt{9ab^3}$ (c) $\dfrac{a\ \frac{2}{b}}{\frac{3}{a}}$ (d) $\dfrac{ab-a}{b^2-b}$ (e) $\dfrac{a^{-1}}{b^{-1}\ \sqrt{a}}$ (f) $\left(\dfrac{a^{\frac{2}{3}}}{b^{\frac{1}{2}}}\right)^2\left(\dfrac{b^{\frac{3}{2}}}{a^{\frac{1}{2}}}\right)$

4. Solve for x (do not use a calculator):

   (a) $5^{x+1}=25$ (b) $\dfrac{1}{3}=3^{2x+2}$ (c) $\log_2 x=3$ (d) $\log_3 x^2=2\log_3 4-4\log_3 5$

5. Simplify: (a) $\log_2 5+\log_2\ \ x^2-1\ \ -\log_2\ \ x-1$ (b) $3^{2\log_3 5}$

6. Simplify: (a) $\log_{10}\ 10^{\frac{1}{2}}$ (b) $\log_{10}\left(\dfrac{1}{10^x}\right)$ (c) $2\log_{10}\sqrt{x}+3\log_{10}x^{\frac{1}{3}}$

7. Solve the following equations for the indicated variables:

   (a) $V=2\ \ ab+bc+ca$, for a (b) $A=P+nrP$, for P

   (c) $2x-2yd=y+xd$, for d (d) $\dfrac{2x}{4\pi}+\dfrac{1-x}{2}=0$, for x

8. Factor completely: (a) x^6-16x^4 (b) $4x^3-8x^2-25x+50$ (c) $8x^3+27$ (d) x^4-1

9. Find <u>all</u> real solutions to: (a) $x^6 - 16x^4 = 0$ (b) $4x^3 - 8x^2 - 25x + 50 = 0$ (c) $8x^3 + 27 = 0$

10. Solve for x: (a) $3\sin^2 x = \cos^2 x;\ 0 \le x < 2\pi$ (b) $\cos^2 x - \sin^2 x = \sin x;\ -\pi < x \le \pi$
(c) $\tan x + \sec x = 2\cos x;\ -\infty < x < \infty$

11. Without using a calculator, evaluate the following:
(a) $\cos 210^0$ (b) $\sin \frac{5\pi}{4}$ (c) $\tan^{-1}\ -1$ (d) $\sin^{-1}\ -1$
(e) $\cos \frac{9\pi}{4}$ (f) $\sin^{-1} \frac{\sqrt{3}}{2}$ (g) $\tan \frac{7\pi}{6}$ (h) $\cos^{-1}\ -1$

12. Sketch the graphs of:
(a) $y = \sin\left(x - \frac{\pi}{4}\right)$ (b) $y = \sin\left(\frac{x}{2}\right)$ (c) $y = 2\sin x$ (d) $y = \cos x$ (e) $y = \frac{1}{\sin x}$

13. Solve the equations: (a) $4x^2 + 12x + 3 = 0$ (b) $2x + 1 = \frac{5}{x+2}$ (c) $\frac{x+1}{x} - \frac{x}{x+1} = 0$

14. Find the remainders after division of:
(a) $x^5 - 4x^4 + x^3 - 7x + 1\ \div\ x + 2$ (b) $x^5 - x^4 + x^3 + 2x^2 - x + 4\ \div\ x^3 + 1$

15. (a) The equation $12x^3 - 23x^2 - 3x + 2 = 0$ has a solution $x = 2$. Find all other solutions.
(b) Solve for x, the equation $12x^3 + 8x^2 - x - 1 = 0$. (All solutions are rational and between ± 1.)

16. Solve the inequalities: (a) $x^2 + 2x - 3 \le 0$ (b) $\frac{2x-1}{3x-2} \le 1$ (c) $x^2 + x + 1 > 0$

17. Solve for x: (a) $|-x + 4| \le 1$ (b) $|5x - 2| = 8$ (c) $|2x + 1| = x + 3$

18. Determine the equations of the following lines:
(a) the line through $-1,3$ and $2,-4$
(b) the line through $-1,2$ and perpendicular to the line $2x - 3y + 5 = 0$
(c) the line through $2,3$ and the midpoint of the line segment from $-1,4$ to $3,2$

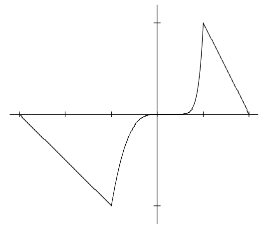
19. (a) Find the point of intersection of the lines: $3x - y - 7 = 0$ and $x + 5y + 3 = 0$.
(b) Shade the region in the xy-plane that is described by the system of inequalities $\begin{cases} 3x - y - 7 < 0 \\ x + 5y + 3 \ge 0 \end{cases}$.

20. (a) Find the domain of the function $f(x) = \frac{3x+1}{\sqrt{x+2}}$.
(b) Find the domain and range of the functions: (*i*) $f(x) = 7$ (*ii*) $g(x) = \frac{5x-3}{2x+1}$

21. (a) Write $f(x)=|x|$ as a piecewise function. (b) Write $f(x)=\dfrac{|x|}{x}$ as a piecewise function.

22. Simplify $\dfrac{f(x+h)-f(x)}{h}$, where (a) $f(x)=2x+3$ (b) $f(x)=\dfrac{1}{x+1}$ (c) $f(x)=x^2$

23. The graph of the function $y=f(x)$ is given to the right:
Sketch the graphs of the functions:
(a) $f(x+1)$ (b) $f(-x)$ (c) $|f(x)|$

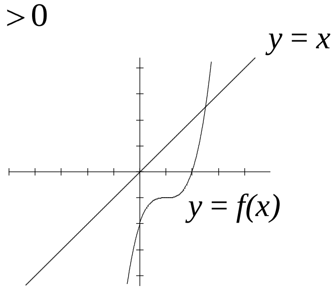


24. Sketch the graphs of the functions:
(a) $g(x)=|3x+2|$
(b) $h(x)=|x(x-1)|$

25. (a) The graph of a quadratic function (a parabola) has x-intercepts -1 and 3 and a range consisting of all numbers less than or equal to 4. Determine an expression for the function.
(b) Sketch the graph of the quadratic function $y=2x^2-4x+3$.

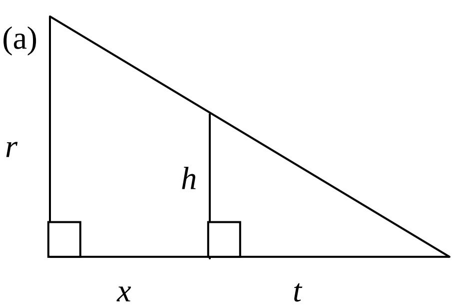
26. Find the inverse of the functions: (a) $f(x)=2x+3$ (b) $f(x)=\dfrac{x+2}{5x-1}$
(c) $f(x)=x^2+2x-1,\ x>0$

27. A function $f(x)$ has the graph to the right.
Sketch the graph of the inverse function $f^{-1}(x)$.

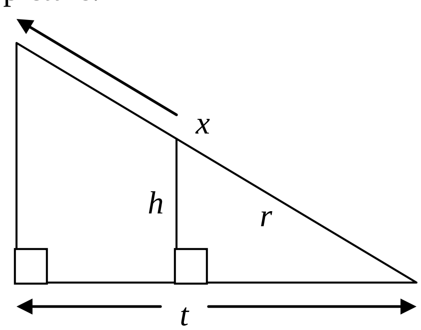


28. Express x in terms of the other variables in the picture.

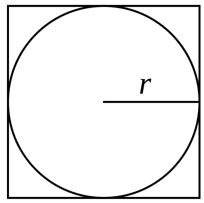
(a)



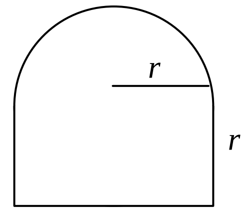
(b)



29. (a) Find the ratio of the area inside the square but outside the circle, to the area of the square in the picture.



(b) Find a formula for the perimeter of a window of the shape in the picture.



(c) A water tank has the shape of a cone (like an ice cream cone without the ice cream). The tank is 10m high and has a radius of 3m at the top. If the water is 5m deep (in the middle) what is the surface area of the top of the water?

(d) Two cars start moving from the same point. One travels south at 100km/hr, the other west at 50 km/hr. How far apart are they two hours later?

(e) A kite is 100m above the ground. If there are 200m of string out, what is the angle between the string and the horizontal? (Assume that the string is perfectly straight.)

30. You should <u>already</u> know the following trigonometric identities.

$(i)\ \sin(-x) = -\sin x$ $\quad (ii)\ \cos(-x) = \cos x$ $\quad (iii)\ \cos(x+y) = \cos x \cos y - \sin x \sin y$

$(iv)\ \sin(x+y) = \sin x \cos y + \cos x \sin y$

<u>Use the above identities</u> to verify the following important identities, which you should also know:

(a) $\sin 2x = 2\sin x \cos x$ (b) $\cos 2x = \cos^2 x - \sin^2 x$ (c) $\cos 2x = 2\cos^2 x - 1$
(d) $\cos 2x = 1 - 2\sin^2 x$

Read through the following notes and examples about the equations of circles and then complete questions 31 through 33.

<u>Equations of Circles</u>

DEFINITION

In geometry, we define a circle as the set of all points $P(x, y)$ in the plane that are equidistant from a given point (center), $C(h, k)$.

We can derive the general equation of a circle using the distance formula. We need to find the distance between $P(x, y)$ and $C(h, k)$ and we call this distance the radius, *r*, of the circle.

$$r = PC$$

$$r = \sqrt{(x-h)^2 + (y-k)^2}$$

$$r^2 = (x-h)^2 + (y-k)^2$$

The general equation of a circle with center at (h,k) and radius r is

$$(x-h)^2 + (y-k)^2 = r^2$$

NOTE: If the center of the circle is at the origin, the equation becomes $x^2 + y^2 = r^2$.

Examples

1) Find the center and radius of the circle: $(x-3)^2 + (y+7)^2 = 19$

Solution:

The center is $(3,-7)$ and the radius is $r = \sqrt{19}$.

2) Find the equation of the circle with center $(7,-3)$ and the circle is tangent to the y-axis.

Solution:

The distance from the center point $(7,-3)$ to the y-axis on the circles edge is 7, this is the radius. Therefore the equation of this circle is $(x-7)^2 + (y+3)^2 = 49$

Side note ➔Remember how to "**complete the square**" from your study of quadratics?

Let's review:

$x^2 + 6x = 4$ (always make sure the constant is on the right)

$x^2 + 6x + \_\_ = 4 + \_\_$ (balance the equation)

$x^2 + 6x + \underline{9} = 4 + \underline{9}$ (take $\frac{1}{2}$ the coefficient of x, square it, and add it to both sides)

$(x+3)^2 = 13$ (factor the left side and simplify the right side)

Now, try this one!

$x^2 + 6x - 1 = 14$ You should be able to get: $(x+3)^2 = 24$

OK, now we need to use the complete the square technique (twice) to find the general form of the equation of a circle in the following example.

3) Find the center and radius of the circle: $x^2 + y^2 - 6x + 4y - 12 = 0$

Solution:

$$x^2 + y^2 - 6x + 4y - 12 = 0$$
$$x^2 - 6x \quad + y^2 + 4y \quad = 12$$
$$x^2 - 6x + \_ \quad + y^2 + 4y + \_ \quad = 12$$
$$x^2 - 6x + 9 \quad + y^2 + 4y + 4 \quad = 12 + 9 + 4$$
$$(x^2 - 6x + 9) + (y^2 + 4y + 4) = 25$$
$$(x-3)^2 + (y+2)^2 = 25$$
$$center \ (3,-2)$$
$$radius = 5$$

Read through the above notes and examples about the equations of circles and then complete questions 31 through 33 below.

----

31. Find the equations of the following circles:
    (a) the circle with center at $1,2$ that passes through the point $-2,-1$
    (b) the circle that passes through the origin and has intercepts equal to 1 and 2 on the *x*- and *y*-axes, respectively.

32. For the circle $x^2 + y^2 + 6x - 4y + 3 = 0$, find:
    (a) the center and radius (b) the equation of the line that is tangent to the circle at the point $-2,5$.

33. A circle is tangent to the *y*-axis at $y = 3$ and has one *x*-intercept at $x = 1$.
    (a) Determine the other *x*-intercept. (b) Deduce the equation of the circle.